

2.1 Relations/Functions

We start with "RELATION". A Relation is a set of ordered-pairs (x,y)

□ A relation where every x-value has no more than one y-value is a FUNCTION

By ordered pairs

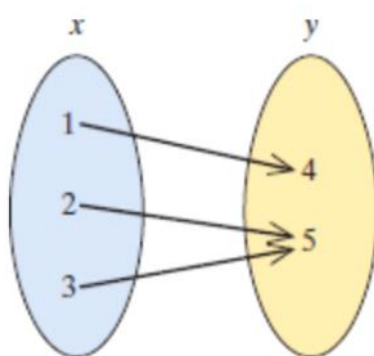
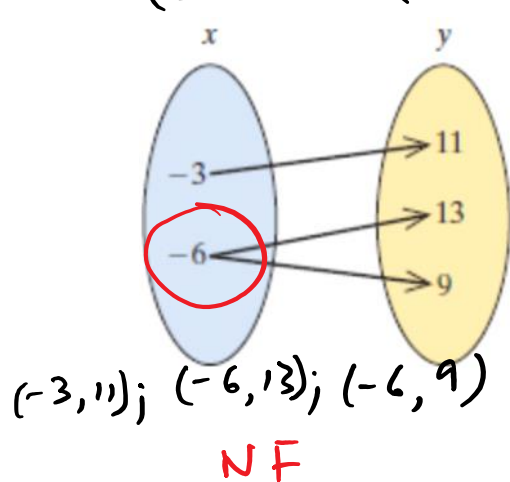
F ✓ (a) $\{(1, 4), (2, 5), (3, 6), (4, 7)\}$

F ✓ (b) $\{(1, 4), (2, 4), (3, 5), (6, 10)\}$

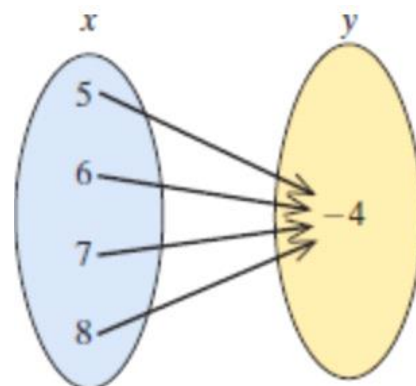
NF (c) $\{(-3, 9), (-2, 4), (0, 0), (1, 1), (-3, 8)\}$

□ mapping

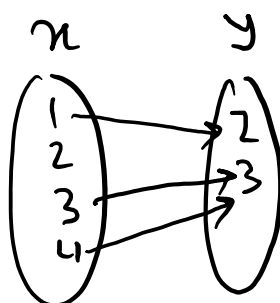
(Domain) (Range)



F



F



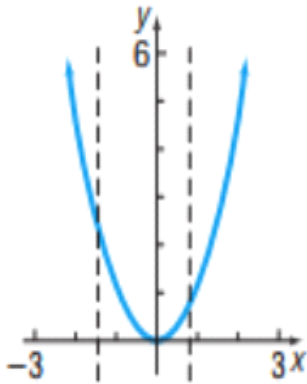
Function

VLT

By graphing

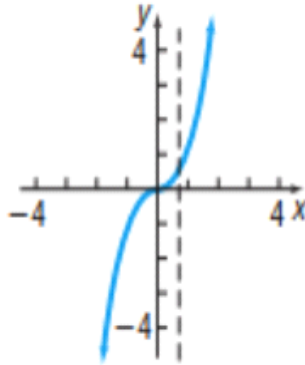
Vertical Line Test

Which of the graphs in Figure 14 are graphs of functions?



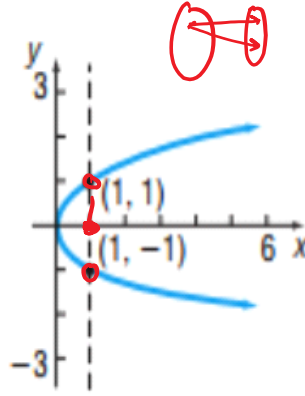
(a) $y = x^2$

F



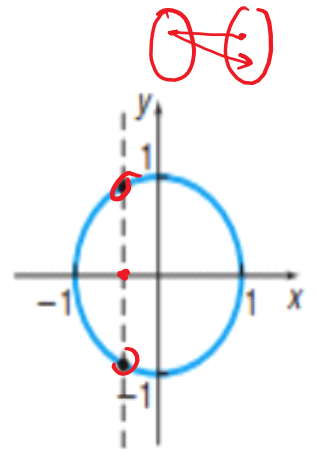
(b) $y = x^3$

F



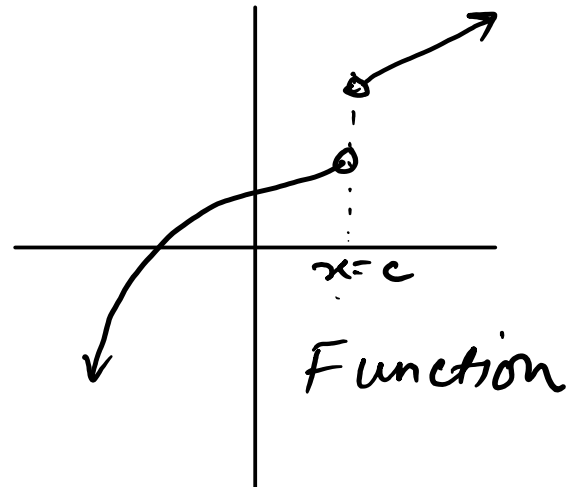
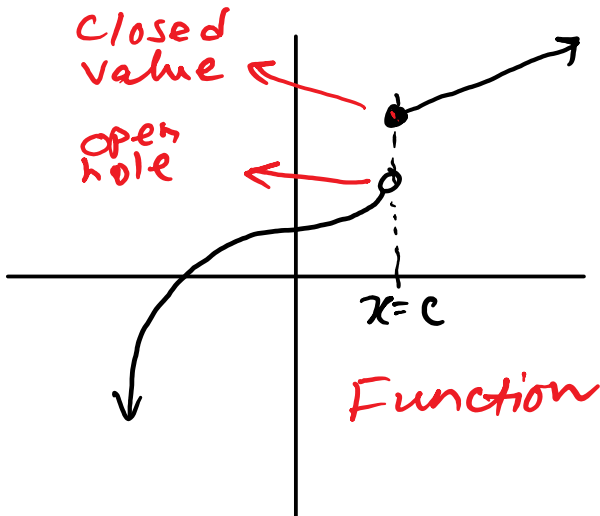
(c) $x = y^2$

NF



(d) $x^2 + y^2 = 1$

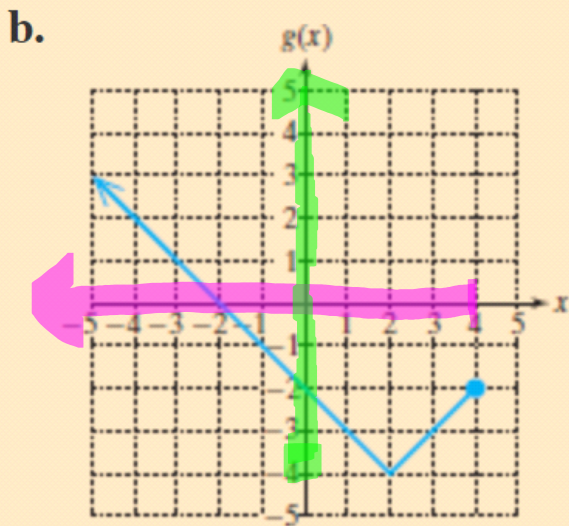
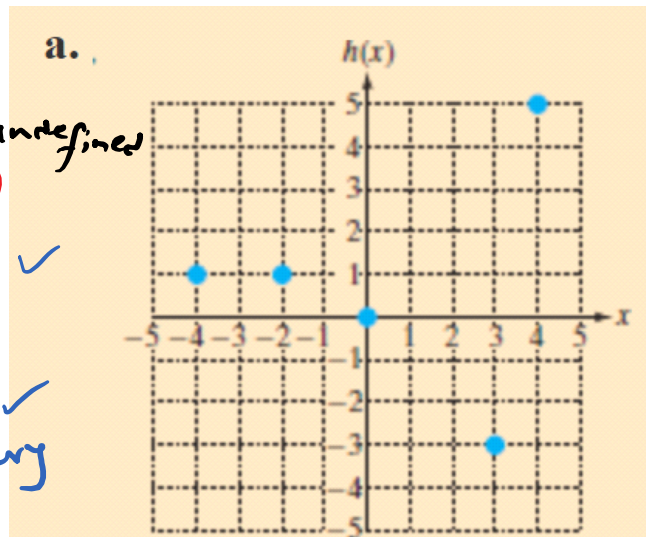
NF



Domain - Range

$$\frac{0}{5} \rightarrow 0$$

$$\frac{5}{0} \rightarrow ? \text{ undefined}$$



Domain: collection of all possible value of x that makes the function defined.

□ Domain: $\{x / x = -4, -2, 0, 2, 4\}$

all x s such that describing

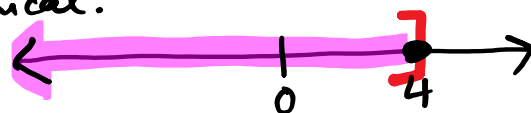
□ Range: $\{y / y = -3, 0, 1, 5\}$

Domain:

① set notation (inequality) $\{x / -\infty < x \leq 4\}$

$\{x / x \leq 4\}$

② graphical:

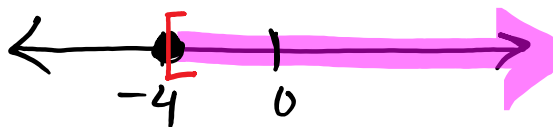


③ Intervals:

$$(-\infty, 4]$$

Range: $\{y / \infty > y \geq -4\}$

$$\{y / -4 \geq y > \infty\} \rightarrow \{y / y \geq -4\}$$



interval: $[-4, \infty)$

2.1 Domain

When is a function defined and undefined?

A function is UNDEFINED at $x = a$ if ANY one of the following occurs...

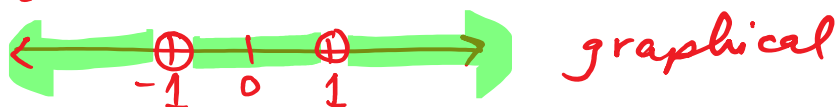
a) $f(a) = \frac{\text{some number}}{0}$ Division by Zero

b) $f(a) = \sqrt{\text{Negative number}}$ Imaginary number

so, how do we write the domain of a function

for 1) $f(x) = \frac{x^2}{x^2 - 1}$

(set denominator $\neq 0$) sol: $x^2 - 1 \neq 0$ (no division by zero)
 $(x-1)(x+1) \neq 0$
 $\downarrow \quad \downarrow$
 $x \neq 1 ; x \neq -1$
Domain: $\{x \mid x \neq \pm 1\}$ set notation or inequality



or $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ Intervals

2.1(NEW)

$\square f(x) = \sqrt{2x-5}$

No negatives inside sq. root
 so, we want $2x-5$ to be greater or equal to 0

$$2x-5 \geq 0$$

$$2x \geq 5$$

$$\left\{ x \mid x \geq \frac{5}{2} \right\}$$

inequality

graphical

interval

$\left[\frac{5}{2}, \infty \right)$

$\square g(x) = \frac{2x}{\sqrt{x-1}}$

→ denominator $\neq 0$
 → no negatives inside $\sqrt{}$

sol: $\sqrt{x-1} > 0$

$x-1 > 0$
 $x > 1$

[not $\sqrt{x-1} \geq 0$ since when $x-1=0$ we will have division by zero]

Domain: $\{ x \mid x > 1 \}$

$(1, \infty)$

$$57) \quad f(x) = \frac{x-2}{x^3+x}$$

$$\text{Domain: } = \frac{x-2}{x(x^2+1)}$$

$$\frac{\#}{0}$$

$\sqrt{-}$

$$x(x^2+1) \neq 0$$

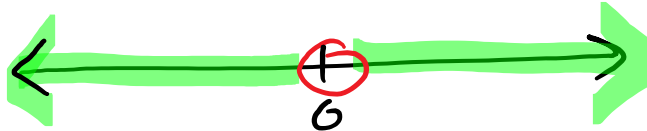
$$\downarrow$$

$$\textcircled{x \neq 0} \quad x^2+1 \neq 0$$

$$\text{Int: } (-\infty, 0) \cup (0, \infty)$$

$$\text{ineq: } \{x \mid x \neq 0\}$$

numb.
line



□ Function Notation and evaluation of function

$y = 2x - 5$
 $f(x) = 2x - 5$
 "f of x" Rule

$y = f(x)$
 value, output dependent
 input independent var.

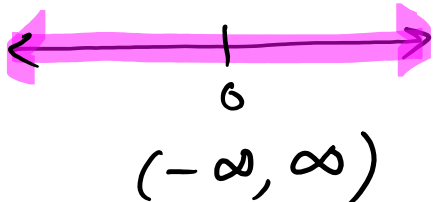
□ $f(x) = 2x^2 - x$

→ $f(0) = 2(0)^2 - (0) = 0 \rightarrow \boxed{(0, 0)}$

→ $f(-5) = 2(-5)^2 - (-5)$
 $= 2(25) + 5$
 $= 50 + 5$
 $= \boxed{55}$
 $\boxed{(-5, 55)}$

→ Domain: $\{x \mid -\infty < x < \infty\} : x \in \mathbb{R}$

x belongs all real numbers



 $(-\infty, \infty)$

DQ-ARC

□ The Difference - Quotient And Average Rate of Change (2.1) (2.3)

given, $y = f(x)$

$$DQ: = \frac{f(x+h) - f(x)}{h}$$

ARC. of $y = f(x)$
in the interval
 $[a, b]$

$$ARC = \frac{f(b) - f(a)}{b - a}$$

ex #1:

$$y = 2x - 5$$

$$f(x) = 2x - 5$$

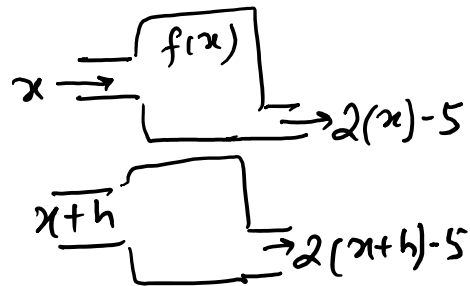
① The DQ: $\frac{f(x+h) - f(x)}{h}$

$$= \frac{[2(x+h) - 5] - [2x - 5]}{h}$$

$$= \frac{2x + 2h - 5 - 2x + 5}{h}$$

$$= \frac{2h}{h}$$

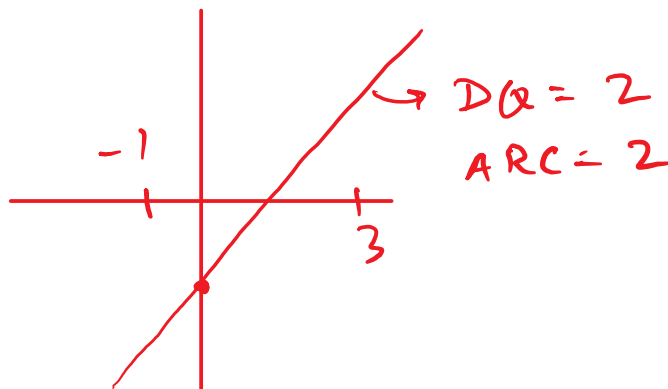
$$= \boxed{2}$$



② Find ARC for $f(x) = 2x - 5$ in $[-1, 3]$

$$\frac{f(b) - f(a)}{b - a} = \frac{[2(3) - 5] - [2(-1) - 5]}{3 - (-1)} = \frac{1 - (-7)}{3 + 1}$$

$$\boxed{2} = \frac{8}{4}$$



Example 2:

$$g(x) = 2x^2 - x$$

① DQ: $\frac{g(x+h) - g(x)}{h}$

$$= \frac{[2(x+h)^2 - (x+h)] - [2x^2 - x]}{h}$$

$$= \frac{[2(x^2 + 2xh + h^2) - x - h] - 2x^2 + x}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h}$$

$$= \frac{4xh + 2h^2 - h}{h} = \frac{h[4x + 2h - 1]}{h} = \boxed{4x + 2h - 1}$$

$$[-1, 3]$$

② ARC: $\frac{f(b) - f(a)}{b - a}$

$$\frac{f(3) - f(-1)}{3 - (-1)}$$

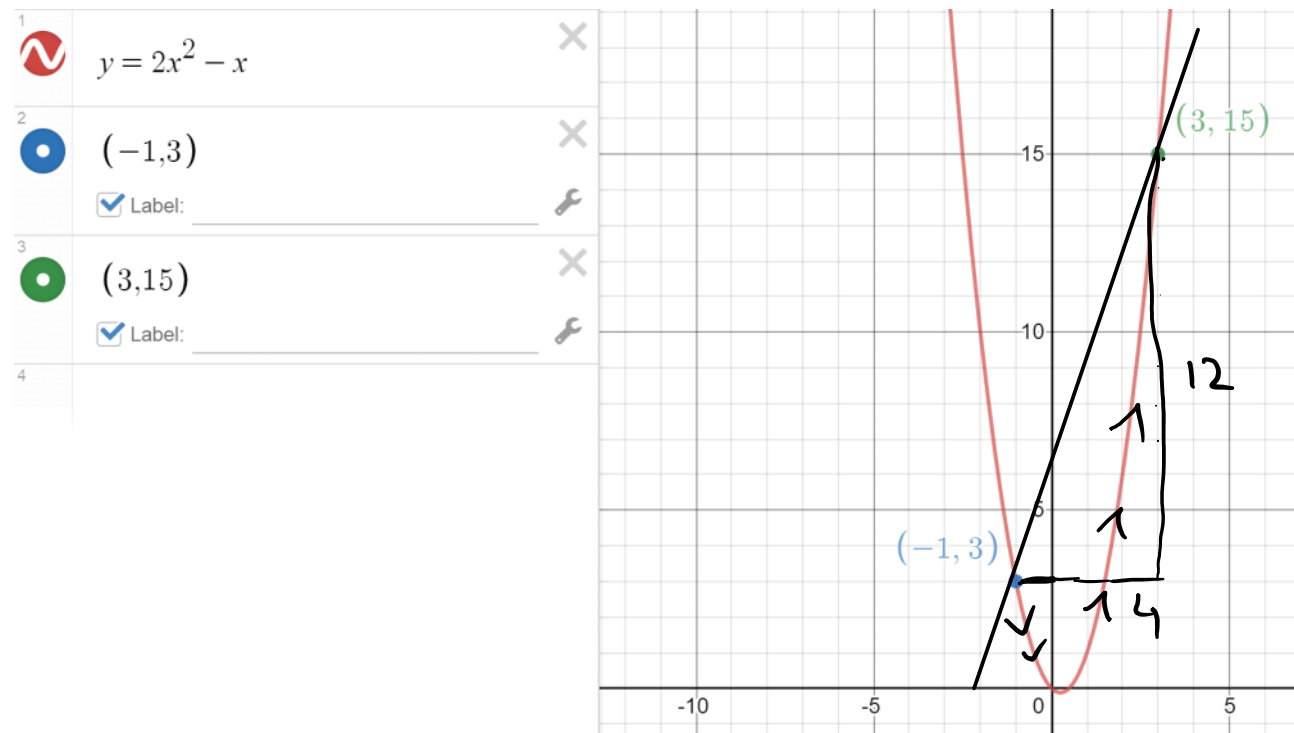
$$\frac{[2(3)^2 - 3] - [2(-1)^2 - (-1)]}{4}$$

$$= \frac{15 - 3}{4} = \frac{12}{4}$$

$$= \boxed{\frac{3}{1}}$$

$$h = \boxed{4x + 2h - 1} \quad |$$

DQ-ARCP3



Example 3

$$f(x) = \frac{1}{\sqrt{x}}$$

$$\text{DQ: } \frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left[\frac{1}{\sqrt{x+h}} \right] - \left[\frac{1}{\sqrt{x}} \right]}{h}$$

$$= \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{\frac{\sqrt{x} - \sqrt{x+h}}{(\sqrt{x+h})(\sqrt{x})}}{h} = \frac{\sqrt{x} - \sqrt{x+h}}{(\sqrt{x})(\sqrt{x+h})} \cdot \frac{1}{h}$$

$$= \boxed{\frac{\sqrt{x} - \sqrt{x+h}}{h[(\sqrt{x})(\sqrt{x+h})]}}$$

$$\sqrt{a} \pm \sqrt{b} \neq \sqrt{a \pm b}$$

$$\sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b}$$

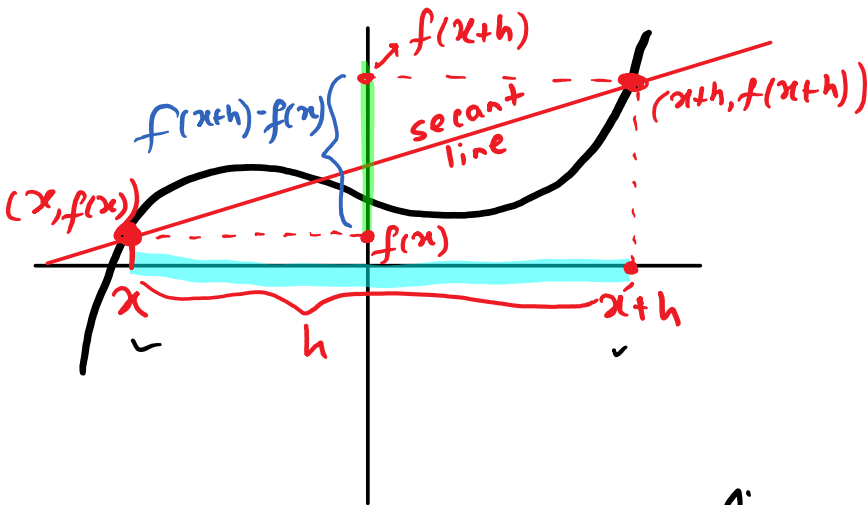
$$a\sqrt{b+c} \neq \sqrt{ab+ac}$$

DQ - ARC connection (NEW)

$$y = f(x)$$

□

Difference Quotient
slope of secant line between
 $(x, f(x))$ and $(x+h, f(x+h))$



DQ = slope of secant line

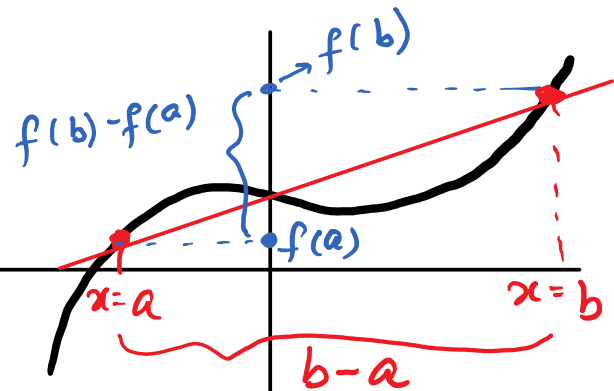
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{f(x+h) - f(x)}{x+h - x}$$

$$= \boxed{\frac{f(x+h) - f(x)}{h}}$$

h = step-size

Average Rate of Change
(slope of secant line
between $(a, f(a))$ and $(b, f(b))$)



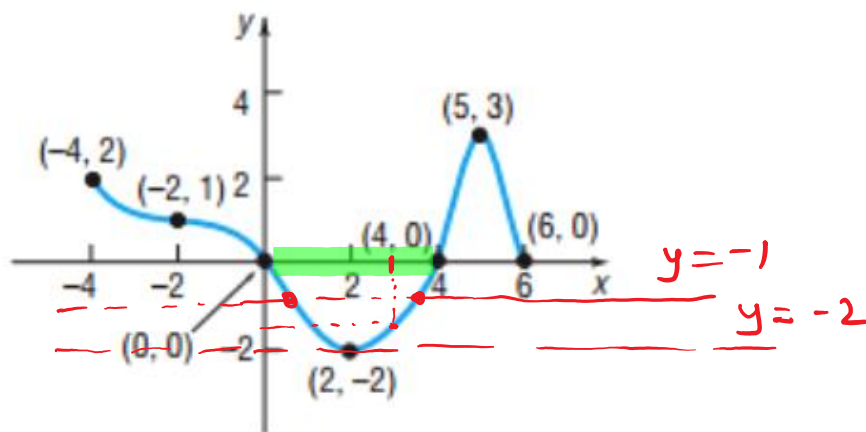
ARC = slope of secant line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \boxed{\frac{f(b) - f(a)}{b - a}}$$

2.2 Analyzing graphs

12. Use the given graph of the function f to answer parts (a)–(n).



- ✓ (a) Find $f(0)$ and $f(6)$. → 0 → 0
- (b) Find $f(2)$ and $f(-2)$.
- ✓ (c) Is $f(3)$ positive or negative? *negative*
- (d) Is $f(-1)$ positive or negative?
- (e) For what values of x is $f(x) = 0$?
- ✓ (f) For what values of x is $f(x) \leq 0$? $(0, 4)$ $0 < x < 4$
 $[0, 4]$ $0 \leq x \leq 4$
- (g) What is the domain of f ?
- (h) What is the range of f ?
- (i) What are the x -intercepts?
- (j) What is the y -intercept?
- ✓ (k) How often does the line $y = -1$ intersect the graph? 2
- (l) How often does the line $x = 1$ intersect the graph?
- (m) For what value of x does $f(x) = 3$?
- ✓ (n) For what value of x does $f(x) = -2$? $x = 2$

$$28. f(x) = \frac{x^2 + 2}{x + 4} \quad f(1) = \frac{(1)^2 + 2}{1 + 4} = \frac{3}{5}$$

✓ (a) Is the point $\left(1, \frac{3}{5}\right)$ on the graph of f ? yes

(b) If $x = 0$, what is $f(x)$? What point is on the graph of f ? $f(0) = \frac{0+2}{0+4} = \frac{2}{4}$

(c) If $f(x) = \frac{1}{2}$, what is x ? What point(s) are on the graph of f ? $\left(0, \frac{1}{2}\right)$ $= \frac{1}{2}$

(d) What is the domain of f ?

✓ (e) List the x -intercepts, if any, of the graph of f .

(f) List the y -intercept, if there is one, of the graph of f .

e) $0 = \frac{x^2 + 2}{x + 4} \rightarrow \underline{x^2 + 2} \neq 0 \rightarrow \underline{\text{No } x\text{-intercept}}$

2.2#37

37. **Cost of Transatlantic Travel** A Boeing 747 crosses the Atlantic Ocean (3000 miles) with an airspeed of 500 miles per hour. The cost C (in dollars) per passenger is given by

$$C(x) = 100 + \frac{x}{10} + \frac{36,000}{x}$$

where x is the groundspeed (airspeed $\pm$ wind).

- What is the cost when the groundspeed is 480 miles per hour? 600 miles per hour?
- Find the domain of C .
- Use a graphing utility to graph the function $C = C(x)$.
- Create a TABLE with TblStart = 0 and $\Delta Tbl = 50$.
- To the nearest 50 miles per hour, what groundspeed minimizes the cost per passenger?

Identify vars: indep. var: groundspeed (miles/hr)

Dep. var: cost/passenger \$/passenger

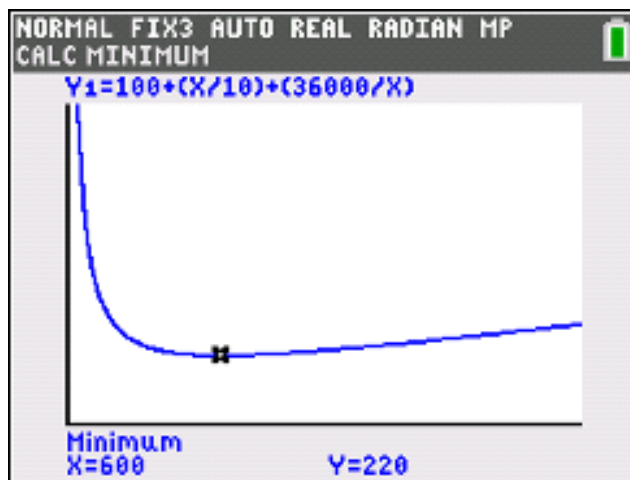
$$\begin{aligned} a) \quad C(480) &= 100 + (480/10) + (36000/480) \\ &= \boxed{\$223.00/\text{passenger}} \end{aligned}$$

$$b) \quad \text{Domain: } \boxed{x > 0}$$

c)

NORMAL FIX3 AUTO REAL Radian MP PRESS + FOR ΔTbl					
X	Y1				
200.00	300.00				
250.00	269.00				
300.00	250.00				
350.00	237.86				
400.00	230.00				
450.00	225.00				
500.00	222.00				
550.00	220.45				
600.00	220.00				
650.00	220.38				
700.00	221.43				

X=700

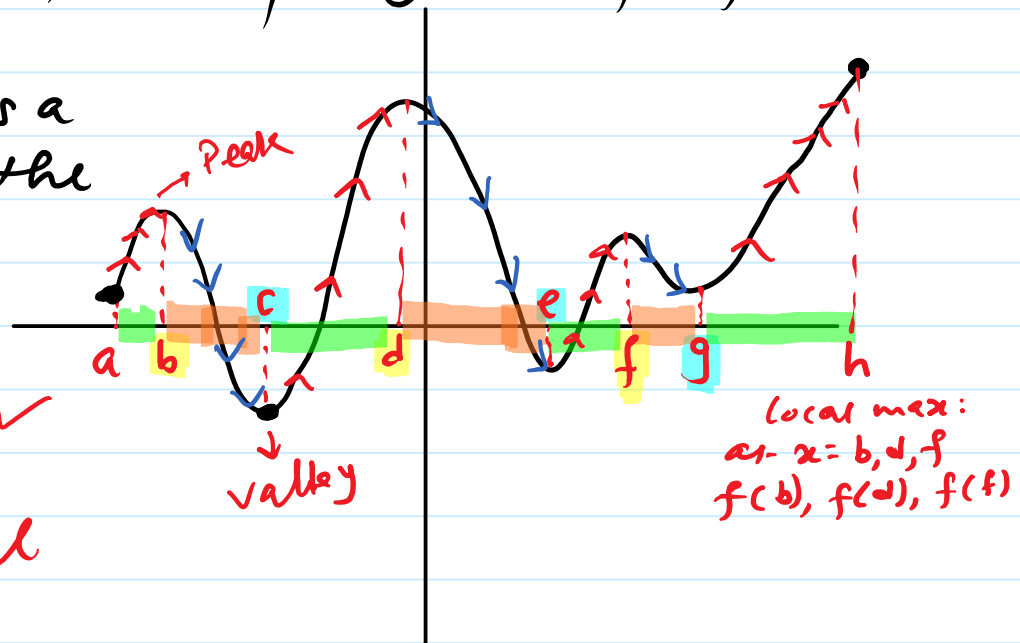


2.3 Local & Absolute extrema

2.2/2.3 Properties of a graph of a function

Suppose,
 $y = f(x)$ is a
 function in the
 window
 $a \leq x \leq h$

or
 $x \in [a, h]$ ✓
 closed
 interval



Extreme Values:

□ Relative max (local max) at
 (needs a "neighborhood") ✓

never at end-point (x) $x = b, d, f$ local max is $f(b); f(d); f(f)$

□ Rel. min (local min) at $x = c, e, g$
 (needs a "neighborhood")

□ Absolute max (can be at end-points
 or interior points)

at $x = h$

□ Abs. min at $x = c$

Intervals where the function is...

□ i) increasing: $(a, b) \cup (c, d) \cup (e, f) \cup (g, h)$
 union

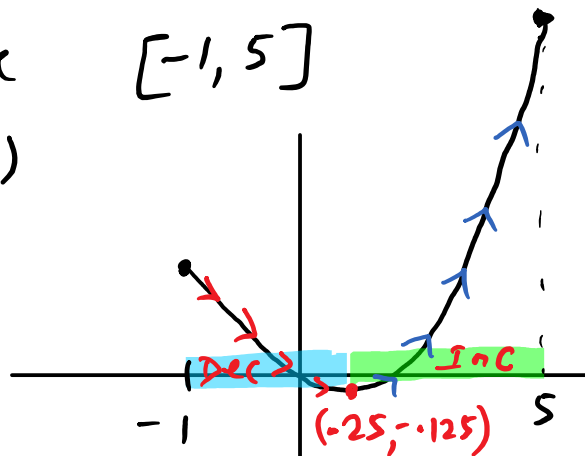
□ ii) decreasing: $(b, c) \cup (d, e) \cup (f, g)$

Inc/Dec p1

$$y = 2x^2 - x \quad [-1, 5]$$

$$f(-1) = 2(-1)^2 - (-1) \\ = 3$$

$$f(5) = 2(5)^2 - (5) \\ = 50 - 5 \\ = 45$$

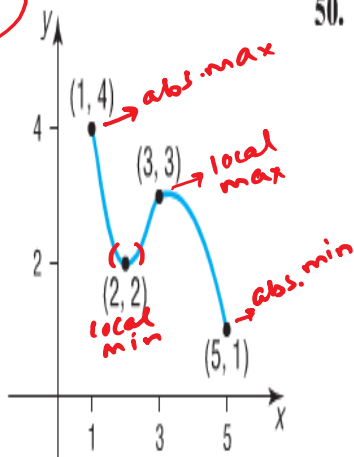


- ① What is the local min: $(-0.25, -0.125)$
- ② What is the local max: None
- ③ Abs. max? $(5, 45)$
- ④ Abs. min? $(-0.25, -0.125)$
- ⑤ Where is $f(x)$ increasing: $(0.25, 5)$ or $[0.25, 5]$
- ⑥ " " " decreasing $[-1, 0.25]$

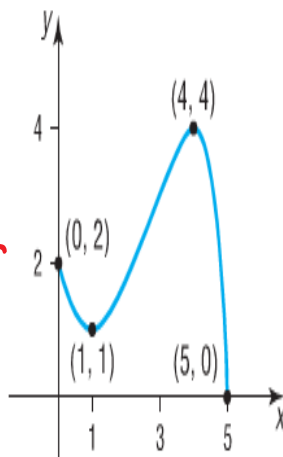
#49-56

In Problems 49–56, for each graph of a function $y = f(x)$, find the absolute maximum and the absolute minimum, if they exist. Identify any local maximum values or local minimum values.

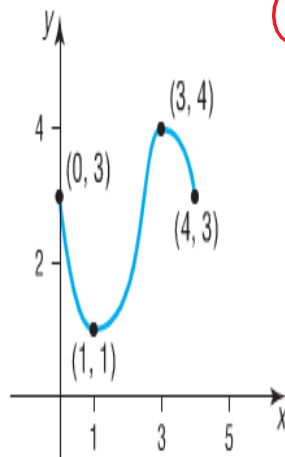
49.



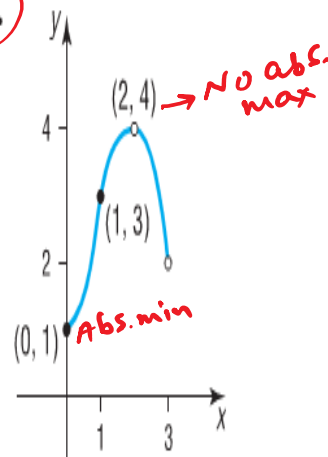
50.



51.



52.



72. $g(x) = x^2 + 1$

- (a) Find the average rate of change from -1 to 2 .
 (b) Find an equation of the secant line containing $(-1, g(-1))$ and $(2, g(2))$.
 (c) Using a graphing utility, draw the graph of g and the secant line obtained in part (b) on the same screen.

$$a) \frac{g(2) - g(-1)}{2 - (-1)} = \frac{[(2)^2 + 1] - [(-1)^2 + 1]}{3} = \frac{5 - 2}{3} = \frac{3}{3} = \boxed{1}$$

b) eq. of secant line:

$$y = mx + b$$

$m = 1$

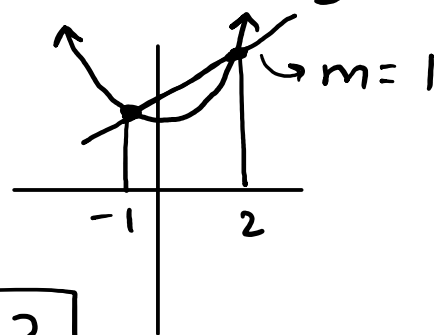
$$y = x + b$$

$$2 = -1 + b$$

$$2 + 1 = b$$

$$\boxed{3 = b}$$

$$\boxed{y = 1x + 3}$$



$(-1, 2)$

